Erasmus Journal for Philosophy and Economics,
Volume 14, Issue 1,
Spring 2021, pp. 210–221.
https://doi.org/10.23941/ejpe.v14i1.607

Mathematical Psychology

Jan Tinbergen

Translated and annotated by:

Conrad Heilmann
Stefan Wintein
Ruth Hinz
Erwin Dekker

Erasmus University Rotterdam

Introduction

'Mathematical psychics' was the name of the approach and the book by Edgeworth for a burgeoning scientific approach, also pioneered by Pareto, for that part of psychology on which economics rests. The nature of the subject of this approach raises the prospect that this approach can also be of interest to practitioners of other sciences related to psychology, which is why an attempt is made here to give an overview of the contents of this approach and some results already achieved with it in economics. In addition, some problems outside economics, narrowly construed, are indicated, for the solution of which one might also make fruitful use of mathematical psychology.

Translators' Note: This article appeared originally in 1930, in Dutch, under the title "Mathematiese Psychologie" in *Mens en Maatschappij*, see Tinbergen (1930). We would like to thank the journal *Mens en Maatschappij* and the Tinbergen estate for their permission to publish the translation. We gratefully acknowledge funding from the research project "Jan Tinbergen: The Thinker", supported by *Stichting Erasmus Trustfunds*, Erasmus Initiative "Dynamics of Inclusive Prosperity", Erasmus School of Economics, Erasmus School of Philosophy, and Erasmus Institute for Philosophy and Economics (EIPE). An accompanying article by Conrad Heilmann and Stefan Wintein that contextualizes the translation appears in the same issue of this journal, see Heilmann and Wintein (2021).

Footnotes with roman numbering appear in the original 1930 article. Annotations by the translators are preceeded by 'Translators' Note'.

I. OUTLINE OF THE MATHEMATICAL-PSYCHOLOGICAL APPROACH[1]

The essence of this approach is that the 'psyche' of an individual can be described by 'functions of equal ophelimity'.* The meaning or intent of this can best be explained by a simple example. The degree of satisfaction of an individual will depend, among other things, on his possession of certain goods. Suppose, for instance, that only two goods X and Y are of interest to the individual. Every possible state is now characterised by the quantities of x and y the individual possesses of those goods. Each such state can be represented graphically by a point (Figure 1), such that $OP_1 = x$ *and* $PP_1 = y$ (x and y being coordinates). A priori, all states are possible that are represented by points within YOX. The curved lines connecting all *those* points representing states which give the individual the same satisfaction, called *ophelimity curves* by Pareto, are important for the behaviour of the individual.

So, when the individual is indifferent whether it is in P_1 or P_2 (that is, possessing either x_1 and y_1 or x_2 and y_2), then P_1 and P_2 lie on the same ophelimity curve. Such curves are, as one can easily see, manifold: there are infinitely many of them. In case both goods are positively valued, the lines will proceed as shown in Figure 2, and the higher lines will then indicate states of greater satisfaction than the lower ones; for it is obvious that, for example, proceeding along the line P_1P, one arrives at states where, besides a constant quantity of x, an increasing quantity of y is possessed. Only one curve goes through each point, because the individual cannot experience two degrees of satisfaction at the same time.

For Pareto, the ophelimity curves serve as a substitute for the utility functions used by other authors. The great advantage of these curves is that they can be experimentally determined and are devoid of any metaphysics. Indeed, one can determine through a survey which combinations of two goods are equally valued by a certain individual. Starting from a certain combination, one can ask, what increase of x can compensate the loss of a unit Y. If these are the quantities x_1, x_2, x_3, x_4, x_5, respectively, then $P_0 \ldots P_5$ is the sought-after curve of constant ophelimity (Figure 3). Of course, these amounts can only be determined within certain limits—thus relatively vaguely—so that one will end up with an area of possibilities instead of lines. But it is clear that the same variation will

* Translators' Note: Tinbergen uses the Paretian term 'ophelimity' for utility. Functions of equal utility later became known as indifference curves.

[1] Cf. the very fine expositions in Bowley's *The Mathematical Groundwork of Economics* (1924); Pareto's *Cours d'Economie Politique* (1896–1897); Lenoir's *Études sur la Formation et le Mouvement des Prix* (1913).

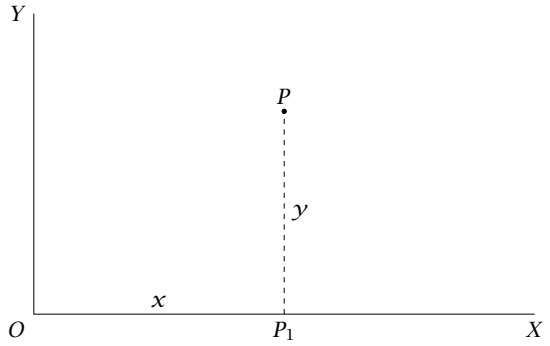

Figure 1

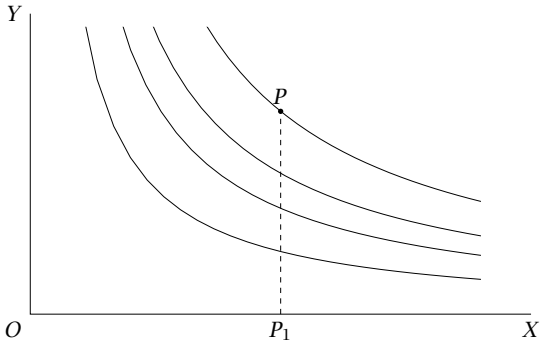

Figure 2

then also occur in the actual actions of people, so that no objection to the depiction of reality lies here.

Thus, one does not ask the question, which in *Pareto's* opinion cannot be answered, of how *large* the value is (the degree of satisfaction, the utility) of a certain combination of goods for the individual. And where it is possible to deduce from these curves the exchanges of goods, which are the foundation of the whole economy of our time, this seems to me a great methodological advantage.

What has been explained here graphically can also be explained algebraically, which is, for cases involving more goods and other parameters that determine the state of the individuals, the most appropriate way. Such a system of lines as described above can be approached by a formula of the form

$$w(x, y) = c$$

where $w(x, y)$ is some function of x and y, while c has a value which is constant for all points of one line, but different for different lines. Here, w is only determined in such a way that any function of w[2] may be substituted for it, that is, the above equation may be replaced by

$$F(w(x, y)) = F(c).$$

When dealing with several goods, w depends on several variables, for example, $w(x, y, z, t)$. This is impossible to represent graphically. For the benefit of the non-mathematically inclined reader, we shall confine ourselves to examples with two variables.

[2] Which fulfils the condition to be not only finite and continuous but also monotonous. For such details, please refer to the literature referenced in footnote 1.

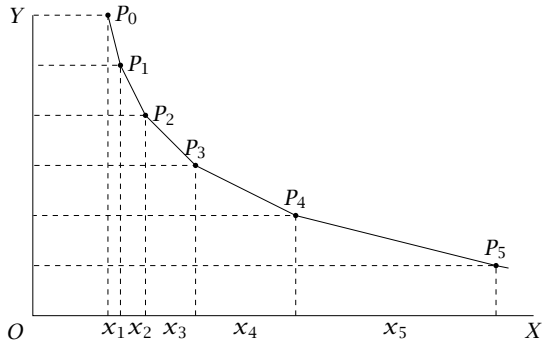

Figure 3

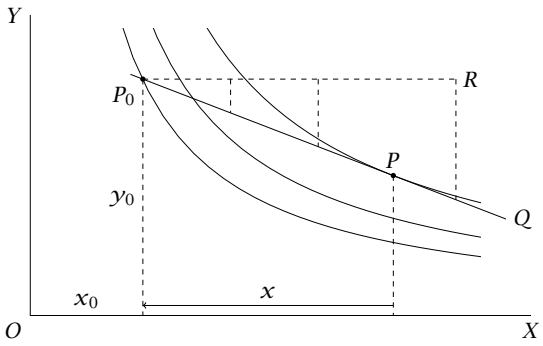

Figure 4

II. RESULTS IN ECONOMICS

From the above diagram of the 'psyche' of economic actors, it is possible to form an idea of their actions in certain circumstances. From the many theoretical-economical inferences for which this scheme allows, we choose a few cases to illustrate.

II.I. Exchange at a Given Price

In economics, exchange means the exchange of property with the consent of each of the relevant individuals—the *homines oeconomici*.

Assume an individual is given the opportunity to exchange, at a given price p, as much Y for X as it chooses. The question is, how much will this individual exchange, when we know how much it originally possesses and when his ophelimity curve is given. This problem I call the problem of exchange at a given price.

Figure 4 illustrates this case, in which the initial state is indicated by the point P_0 (the individual is endowed with x_0, y_0). The opportunity to exchange at a given price p can be represented graphically by allowing the individual to move its property combination anywhere along the line P_0Q, which is drawn so that $p = {}^{RQ}/_{P_0R}$. The individual will now choose, among the given possibilities, the one that yields the most satisfaction. And that is the situation, denoted by P, where the line P_0Q intersects the highest ophelimity curve: every other point of P_0Q lies on a lower ophelimity curve. The quantity x (see Figure 4) which the individual will want to exchange is called in economics the demand at this price (or, if negative, the supply).

With the help of the above, one can now construct a theory of exchanges between many individuals among themselves, each of which considers the price to be fixed. At any conceivable price, which could prevail in such a 'market', the supply and demand of each of the individuals is

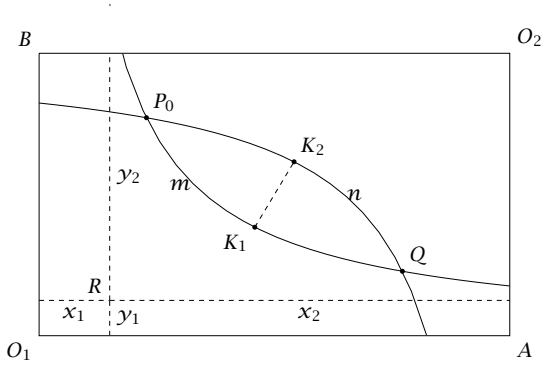

Figure 5

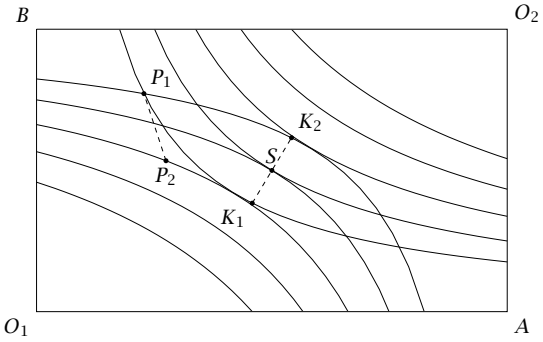

Figure 6

fully determined, and the actual ('equilibrium') price will now be such that total demand and total supply are equal.

II.II. Isolated Exchange

By isolated exchange we understand an exchange where there is one individual on each side, in contrast to the exchange at fixed price discussed above. In the case of isolated exchange, there is, as a matter of principle, no competition. There is no price which each of the individuals considers given. It now appears that the method of mathematical psychology clearly demonstrates what the most important peculiarity of this exchange is: without a further organising principle this exchange has no fixed outcome; through economic forces alone no equilibrium is established. Figure 5 illustrates this. The possibilities in this case can be represented very clearly in the manner shown in this figure: the rectangle O_1AO_2B has as sides, respectively, O_1A = the sum total of property X owned by both individuals and O_1B = the sum total of property Y owned by both individuals. All possible distributions of that total property—that is, all possibilities after the exchange—can now be represented by points within this rectangle. An arbitrary point R therein may, by its distance from O_1B, O_1A, O_2A, and O_2B, respectively, indicate the possession of X and Y by individual 1 and 2 (x_1, y_1, x_2, y_2). In this rectangle, we can now draw the ophelimity curves: for individual 1, relative to the axes O_1A and O_1B as x- and y-axes, and for individual 2, relative to O_2B and O_2A as x- and y-axes (for example, m and n). If the rectangle is thus covered with a net of ophelimity curves (Figure 6), such that through each point there pass two, one from each individual, it is possible to deduce for each change what either of the individuals think about it. Moving from P_1 to P_2 will be gladly accepted by individual 2, as he moves to a higher ophelimity curve. On the other hand, 1 will not desire making this step. Since an exchange

always requires the consent of both parties, from a given starting point P_0 (Figure 5), only those points which are situated between the ophelimity curves of both parties, m and n (as they are drawn through P_0), are possible as endpoints of the exchange. The endpoint can thus only be in the lens-shaped area P_0mQn. We can specify the endpoint a little further. For it shall not be possible that either individual can still gain an advantage. This means, as can easily be seen, that from the endpoint one cannot draw anew a lens-shaped area, as was the case from P_0. As a consequence, only points where the ophelimity curves of both individuals *meet*, as in point S, Figure 6, can serve as endpoints. There is a whole series of such points, forming a line, which is usually called a *contract curve* (K_1K_2 in Figure 5 and Figure 6).

Therefore, all points on the line K_1K_2, in so far as they lie within the lens P_0mQn, can function as end points of the exchange. Without further information, it is not possible to determine which of these points is preferable. Thus, the exchange result is indeterminate within certain limits.[3]

A number of similar issues, in which the consequences of a certain market organisation for the determination and level of the price are examined, find a clear formulation in the mathematical-psychological way of thinking. One thinks here of cases like imperfect competition.

III. Mathematical Psychology as a Starting Point for More Concrete Research

I would now like to indicate a few directions in which, to my mind, expansion of the mathematical-psychological approach could be of interest.

In the first place, it is clear that there is much room for experimental research. Among other things, attention will have to be paid not so much to the ophelimity curves for individual goods as to those for groups of goods (food, clothing, housing, recreation, study) against each other. Apart from the actual shape of the ophelimity curves, which can be expressed numerically, it is very interesting to ask whether the ophelimity curves differ greatly between persons, and to what extent these differences are due to characteristics that can be easily identified (illness, profession). Here lies an area of psychology which, if I am well informed, is still open to considerable expansion.

[3] The importance of these questions is especially pointed out by *Schumpeter* in his study "The Instability of Capitalism" (1928); it is true that Schumpeter arrives at a different conclusion as far as the question raised here is concerned; however, his clear statement of the problem is only possible through the use of the mathematical-psychological method.

Certainly, some such research already exists. I am thinking here of the meritorious budget studies of various municipal statistics offices. However, in their usual form, these do not provide enough data to learn about the ophelimity curves.[4]

Secondly, there is an extensive possibility to further accentuate the considerations given in section II and similar ones, by working with special cases of w. One of the most important forms for w is: the profit of an enterprise.

IV. QUESTIONS OF JUSTICE; FORMULATION OF A CRITERION OF 'JUST DISTRIBUTION'

In addition to these purely economic applications, the mathematical-psychological method seems to be useful for the analysis of the common conception of justice with regard to the rules governing the distribution of economic goods.[5] This analysis, which concerns one of the oldest societal problems, has recently come to the fore again, often even in a very practical, quantitative form, for example, because of the increasing organisation of society, as a result of which distribution through free competition is no longer possible and the need for a separate distribution principle has come to the fore.

The determination of 'just distributions', for example, the distribution of the total production among all individuals, must of course be preceded by a definition of justice. As far as I can see, the common conception of justice[†] entails the equal treatment of equally situated individuals. This criterion then provides a solution, for example, in the following simple case:

A) Two identical workers perform the same amount of labour of a certain kind. Which distribution of the product can be considered just? The criterion mentioned above gives the solution of equal distribution: equal

[†] Translators' Note: Tinbergen uses *het rechtsbewustzijn*, which alludes to a (shared) sense of justice; it is not clear from the text whether he believes it is shared by all.

[4] They give the quantities bought of different commodity groups in relation to income. From some of these investigations it can be concluded that a quadratic form of w gives a sufficient approximation; of the n^2 unknown coefficients (n being the number of commodity groups) only n combinations can be calculated.

At this point, it is worth mentioning the interesting idea of Irving Fisher (1927), developed in the Festschrift for Clark, to determine the marginal value of money in relation to income in such a way.

[5] These problems cannot be considered part of pure economics today; according to *Sombart*'s terminology in *Die Drei Nationalökonomien* (1930, 295), they belong partly to "economic ethics" ["Wirtschaftsethik"], partly to "the art of economics" ["Wirtschaftskunstlehre"]. The following problems and solutions were developed in very animated discussions with Prof. Ehrenfest.

wages. However, it cannot provide a solution in problems involving individuals in *unequal* circumstances, as, for example, when one extends the above case A in one or more of the following ways:

1) the quantities of work are unequal;
2) the workers have different ophelimity curves;[6]
3) the type of work is different.

As soon as one of these cases arises, in which the circumstances (in the broadest sense) of the individuals are different, the possibility of applying the principle of equal treatment is no longer there. The question is, how to extend the criterion of just distribution to these cases. In my opinion, the requirement *that the individuals should not have the desire to occupy each other's position (in the broadest sense)* should be adopted as such. We postulate this answer to the above question without any further justification other than that the first-mentioned 'simple justice criterion' must be included as a special case, which is indeed the case. Thus, we leave a discussion of the correctness of our concept of justice to the relevant science (in the broader sense of *Lehre*[‡]).

The more general criterion we formulated can be represented very conveniently in the language of mathematical psychology. Is the position (in the broadest sense) of individual 1 indicated by the quantities x_1, y_1, z_1 (in our example, we denote with x the amount of work type 1, y amount of work type 2, and with z wages) and by the function w_1, and likewise the position of individual 2 by x_2, y_2, z_2, and w_2, then it reads:

$$w_1(x_2, y_2, z_2) \leqq w_1(x_1, y_1, z_1) \qquad w_2(x_1, y_1, z_1) \leqq w_2(x_2, y_2, z_2)$$

For the first equation expresses that the circumstances of individual 2 do not give greater satisfaction to 1 than his own circumstances; the second that the circumstances of individual 1 do not give greater satisfaction to 2 than his own position.

For cases in which there are only two dimensions, such as when our case A is only expanded according to 1 or 2, one can again use the graphical representation.

[‡] Translators' Note: This is most likely a reference to the art of economics as referenced in footnote 5.

[6] An open question for me here is whether it is sufficient for the psychological 'identity criterion' to hold for two people to be equal in terms of their ophelimity curves.

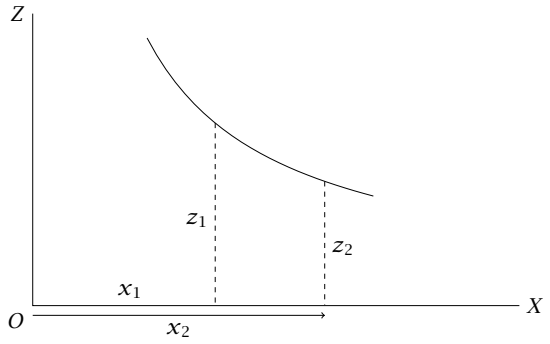

Figure 7

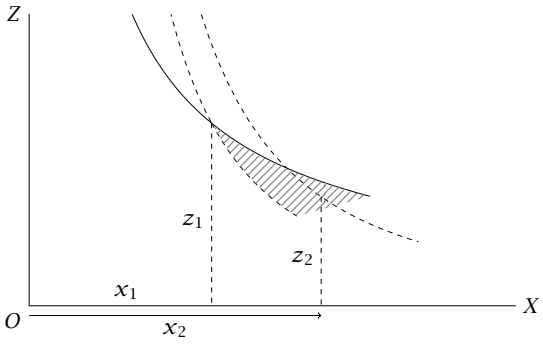

Figure 8

A1) So here w_1 and w_2 are the same functions, while x_1 and x_2 are unequal; our criterion becomes:

$$w(x_2, z_2) \leqq w(x_1, z_1) \qquad w(x_1, z_1) \leqq w(x_2, z_2)$$

which can only be satisfied simultaneously by the equality signs; the meaning of this is, that z_1 and z_2 will be such that the points x_1z_1 and x_2z_2 lie on one ophelimity curve. Thus z_2 is determined by x_1, x_2, and z_1 (Figure 7).

A2) The workers do not have the same ophelimity curves. If, however, they do the same amount of work, the 'just distribution' will still be equal pay; if, however, also the amounts of work are different, our criterion is:

$$w_1(x_2, z_2) \leqq w_1(x_1, z_1) \qquad w_2(x_1, z_1) \leqq w_2(x_2, z_2)$$

The two points x_1z_1 and x_2z_2 must now be situated such that x_2z_2 lies below the ophelimity curve of individual 1 through x_1z_1 (the line in Figure 8) and at the same time x_1z_1 lies below the ophelimity curve of individual 2 through x_2z_2 (the dotted line).

The relevant difference with case A1 is that, for given x_1, z_1, and, for example, x_2, z_2 still has a certain degree of freedom, or that, for given x_1 and z_1, x_2 and z_2, the point x_2z_2 can still lie in the shaded area.[§]

The examples provide sufficient clarity of the criterion of justice posited here.

It seems to me that this train of thought can be extended in two directions. Firstly, under what conditions does a 'just distribution' automati-

[§] Translators' Note: For a more extensive description of Figure 8, see Heilmann and Wintein (2021, 230).

cally come about, and secondly, when should one consciously strive for such a distribution.

V. AUTOMATIC ACHIEVEMENT OF 'JUST DISTRIBUTIONS'; IMPEDIMENTS AGAINST AUTOMATIC ACHIEVEMENT

According to our criterion, a just distribution exists when any individual can indeed, if so desired, reach a state (in the broadest sense) equal to that of any other individual: when there is 'free mobility' in all respects.[7] With respect to wages and labour performance, this is the case, for instance, with free competition, infinitely easy mobility, and 'retraining', etc. Amongst workers, an approximation of justice is then achieved.

Where, however, the aforementioned freedom of movement, taken as broadly as possible, is lacking, it is not guaranteed that a 'just distribution' is reached. This freedom of movement can be absent in very many ways, due to very different causes. We mention some examples:

1. Levelling the living conditions of people in different geographical areas is still mostly impossible due to the lack of free migration (legal, physical, and psychological barriers). Many and large injustices still exist in this area: for example, the difference in real wages for the same work in the United States, the Netherlands, Germany, Austria, Russia, and China.

2. A person cannot, to a large extent, exchange his own state of health with that of others. Great injustice still exists, though to a much lesser degree than before, between the sick and the healthy. Something similar existed and still exists with regard to the size of the family.

3. Freedom of movement from one profession to another is significantly impeded for all skilled professions by the duration and costs of education.[8] This duration and costs are an absolute impediment for important groups of people to learn certain trades. In this respect, the greater or lesser degree of aptitude, which in many cases has an important influence, does not, in my opinion, justify a higher income, just as poor health cannot justify a lower income.

4. Another impediment to freedom of movement is the inequality of property, as a result of the private ownership of the means of production and inheritance law. In the opinion of many, these injustices lie at the heart of the class contradictions of our time.

[7] As the Liberal Socialists, led by Oppenheimer, demanded in contrast to the other Socialist reformers.

[8] If an individual once studies a trade and is disappointed with it later on (here, however, another element plays a role, that is, not knowing one's own ophelimity function properly or changing it), this too can lead to an unjust distribution later on.

5. An interesting cause of reduced freedom of movement is also the economic organisation of society, such as on the one hand the formation of trusts etc., and on the other hand trade unions. The peculiar seclusion of certain trades, which occurs more and more and of which the final and most serious consequence is unemployment, also opens the possibility of important injustices in the distribution of the incomes.

Amongst the obstacles to free movement mentioned, there are some that could be eliminated by human effort if desired; there are others, such as state of health and ability, that certainly cannot be eliminated. In such cases, therefore, it makes sense to strive consciously and directly for a just distribution of 'benefits and burdens'. This is done through various social institutions and laws. However, the quantitative yardsticks applied are for the time being rather arbitrary and, consequently, the opinions of different people on these numbers fluctuate widely. Examples of quantitative indeterminacy are sickness benefits, child allowances, accident benefits, unemployment benefits (on which the views vary between full pay and nil), holiday allowances, and widows' and orphans' pensions. In my opinion, the method of mathematical psychology outlined above can contribute to the solution of these contested issues.

The method described above may also be useful in cases where freedom of movement could be restored through human action but where this is not considered desirable for other reasons, such as unjust distribution as a result of the increasing organisation of society.

For other, larger questions, such as the question whether inheritance laws are in accordance with the common conception of justice, one will first have to consider, as is done in legal science, the complications of the questions mentioned, such as in this case, for example, the question to what extent one should consider a person as one whole with his ancestors or as a completely independent individual. Difficulties of this kind are those arising from the changeability of w, whereby a distribution originally in accordance with our criterion of justice may later no longer be in accordance with it. Another, very interesting set of questions, which, however, seems to me for the time being to be only of academic interest, concerns the question: to what extent does exchange make an originally just distribution of goods unjust or vice versa?

However, it would take us too far to go into the issues mentioned in this last paragraph.

REFERENCES

Bowley, Arthur L. 1924. *The Mathematical Groundwork of Economics: An Introductory Treatise.* Oxford: Clarendon Press.

Fisher, Irving. 1927. "A Statistical Method for Measuring 'Marginal Utility' and Testing the Justice of a Progressive Income Tax." In *Economic Essays Contributed in Honor of John Bates Clark*, edited by Jacob H. Hollander, 157–193. New York, NY: The Macmillian Company.

Heilmann, Conrad, and Stefan Wintein. 2021. "No Envy: Jan Tinbergen on Fairness." *Erasmus Journal for Philosophy and Economics* 14 (1): 222–245.

Lenoir, Marcel. 1913. *Études sur la Formation et le Mouvement des Prix*. Paris: M. Giard et E. Brière.

Pareto, Vilfredo. 1896–1897. *Cours d'Économie Politique*. 2 vols. Lausanne: F. Rouge.

Schumpeter, Joseph. 1928. "The Instability of Capitalism." *The Economic Journal* 38 (151): 361–386.

Sombart, Werner. 1930. *Die Drei Nationalökonomien*. Munich and Leipzig: Duncker and Humblot.

Tinbergen, Jan. 1930. "Mathematiese Psychologie." *Mens en Maatschappij* 6 (4): 342–352.

Conrad Heilmann is associate professor of philosophy and co-director of the Erasmus Institute for Philosophy and Economics (EIPE) in the Erasmus School of Philosophy and Core Faculty of the Erasmus Initiative "Dynamics of Inclusive Prosperity" at Erasmus University Rotterdam. He is the co-editor (with Julian Reiss) of *The Routledge Handbook of the Philosophy of Economics*.
Contact e-mail: <heilmann@esphil.eur.nl>

Stefan Wintein is assistant professor of theoretical philosophy at the Erasmus School of Philosophy (ESPhil) and member of the Erasmus Institute for Philosophy and Economics (EIPE), both at Erasmus University Rotterdam in the Netherlands. Together with Conrad Heilmann, he runs the Fairness project: http://fairness-research.org.
Contact e-mail: <wintein@esphil.eur.nl>

Ruth Hinz is a research master's student in philosophy and economics at EIPE, Erasmus University Rotterdam. Her research interest focuses on fairness; in particular, on fairness in questions related to social choice theory.
Contact e-mail: <hinz@esphil.eur.nl>

Erwin Dekker (1984) is assistant professor of Cultural Economics at the Erasmus University Rotterdam and has been post-doctoral fellow at the Mercatus Center at George Mason University. He has recently published *Jan Tinbergen (1903–1994) and the Rise of Economic Expertise* (2021) and previously *The Viennese Students of Civilization: The Meaning and Context of Austrian Economics Reconsidered* (2016), both with Cambridge University Press.
Contact e-mail: <e.dekker@eshcc.eur.nl>