

# No Envy: Jan Tinbergen on Fairness

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**Abstract:** The important ‘no-envy’ fairness criterion has typically been attributed to Foley (1967) and sometimes to Tinbergen (1946, 1953). We reveal that Jan Tinbergen introduced ‘no-envy’ as a fairness criterion in his article “*Mathematische Psychologie*” published in 1930 in the Dutch journal *Mens en Maatschappij* and translated as “Mathematical Psychology” in 2021 in the *Erasmus Journal for Philosophy and Economics*. Our article accompanies the translation: we introduce Tinbergen’s 1930 formulation of the ‘no-envy’ criterion, compare it to other formulations, and comment on its significance for the fairness literature in philosophy and economics.

**Keywords:** no envy, subjective equity, equity, fair, fairness, fair division

**JEL Classification:** B21, D63

## I. INTRODUCTION

The economist Jan Tinbergen—along with Ragnar Frisch, the recipient of the inaugural 1969 Nobel prize in economics—is not only widely regarded as an econometrician and engaged scientific expert, but also as a theorist of redistribution.<sup>1</sup> In this article, we reveal that Jan Tinbergen already

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<sup>1</sup> See Dekker (2021) who develops these themes, in particular the topic of expertise, in the first intellectual biography of Jan Tinbergen.

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introduced the ‘no-envy’ criterion in print in 1930, compare it to other formulations, and comment on the significance of his article for the fairness literature in philosophy and economics.

The ‘no-envy’ criterion has been a cornerstone of many influential theories of fairness for a few decades now (Thomson 2011). It characterizes a state of affairs as ‘envy-free’ if no individual prefers another individual’s allotment to their own. Arnsperger (1994, 155) characterizes it as a fundamental criterion of distributive justice that is “absolutely central” for economic theory. So far, the ‘no-envy’ criterion has typically been attributed to three sources: firstly, and most commonly, Foley’s (1967) article is cited. Secondly, the two editions of Jan Tinbergen’s book *Redelijke Inkomensverdeling* (1946, 1953) are sometimes also cited alongside it. Thirdly, there are occasional references to conversations between Jan Tinbergen and his supervisor, Paul Ehrenfest, in the early 1920s (such as in Kolm [1971] 1997; Young 1994; Lambert 2012; Olson 2018). In this article, we demonstrate that an additional attribution of the ‘no-envy’ principle to Jan Tinbergen is in order: the ‘no-envy’ principle appears in his 1930 article “Mathematische Psychologie” (Mathematical Psychology), published in the Dutch journal *Mens en Maatschappij* and translated in 2021 in the *Erasmus Journal for Philosophy and Economics* (Tinbergen [1930] 2021). There are thus not three, but at least four attributions relevant for the origins of the modern ‘no-envy’ principle. As far as we know, the 1930 article is the first appearance of the ‘no-envy’ criterion in print.

Tinbergen’s 1930 article discusses the essential ingredients of early welfare economics as developed by Edgeworth and Pareto before presenting the ‘no-envy’ criterion. While perhaps of historical interest in the context of welfare economics, we leave aside a more detailed summary and discussion of these elements of the article, in order to focus on Tinbergen’s introduction of the ‘no-envy’ criterion. We discuss similarities and differences between how Tinbergen discusses the ‘no-envy’ principle in his 1930 article and his 1946/1953 book. We also compare it to Foley’s (1967) formulation, as the latter has so far been the most prominent source of ‘no-envy’. We will show that while all three sources present essentially the same ‘no-envy’ criterion, the issue of what the criterion is applied to is more ambiguous. On the one hand, both authors speak of the criterion being applied to an individual’s ‘place’ or ‘position’ within society. On the other hand, they also propose different precise objects to which the ‘no-envy’ criterion applies: Tinbergen (1930) considered hours worked and wages earned and Tinbergen (1946, 1953) considered income distribution (on the basis of hours worked and wages earned), whereas Foley (1967)

considered consumption bundles. Interestingly, Tinbergen's early formulation of the 'no-envy' principle is in some ways more concrete than the later discussions by both himself and Foley. In addition to our discussion of Tinbergen and Foley on 'no-envy', we also briefly comment on more recent contributions to the 'no-envy' literature, and we make two general observations about the relations between Tinbergen's contributions and conceptual discussions of fairness.

We proceed as follows. Section II introduces the 'no-envy' criterion and presents Tinbergen's (1930) and (1946, 1953) formulations of it. Section III compares Tinbergen with Foley (1967) and discusses both in relation to other contributions in the 'no-envy' literature. Section IV makes the two general observations on conceptual discussions of fairness. Section V concludes.

## II. INTRODUCING 'NO-ENVY'

### *III. The Origins of the Modern 'No-Envy' Criterion*

If Alice does not prefer Bob's allotment to her own, and Bob does not prefer Alice's allotment to his own, the situation they find themselves in is 'envy-free'. The idea of no individual preferring another individual's allotment over theirs is typically referred to as the criterion of 'no-envy' or 'envy-freeness'. This criterion is used in two different, but closely related, ways. Firstly, the 'no-envy' criterion is used to favour some allocations, distributions, or divisions of goods, or indeed whole states of social affairs over others, as they are 'fairer' (Arnsperger 1994; Thomson 2011). Secondly, many theorists see an absence of envy between individuals as important for, if not defining of, the *concept* of fairness. They explore in how far the 'no-envy' criterion is a constitutive aspect of the concept of fairness in general (Olson 2018; Fleurbaey 2021).

Philosophers and economists alike emphasize that 'no-envy' is supposed to be a technical concept that is independent of sentiment (Kolm 1996; D'Arms 2017; Olson 2018). Envy, as encapsulated in the 'no-envy' criterion, does not point to the presence of a feeling of resentment or jealousy, or to any psychological fact about the individuals involved that goes beyond the following: whether they prefer what the other has over what they have themselves. In short, 'no-envy' is just a commonly agreed upon label for a state of affairs in which individuals do not prefer to have what other individuals have. It is independent of whether individuals do actually experience any envy.<sup>2</sup> Preference-based notions of 'no-envy' have

<sup>2</sup> For a more detailed defence of the 'impersonal' nature of the preference-based 'no-envy' concept, see Olson (2018).

become key for many theories of fairness, reviewed in Thomson (2011) and Fleurbaey (2021, section 6).

While fairness theories that are based on ‘no-envy’ do not rely on psychological content beyond preferences,<sup>3</sup> they are nevertheless to be classified as *subjective* fairness theories. They locate the meaning of fairness in the subjective judgement of the individuals who are concerned with the social state of affairs in question. In this sense, ‘no-envy’ based fairness theories are thus subjective, even if they do not rely on a broader psychological characterisation of the individuals concerned. Such subjective theorizing about fairness is to be distinguished from objective (or non-subjective) fairness theories in philosophy and economics.<sup>4</sup> To appreciate the difference between subjective and objective fairness theories, consider the following example.

**Owing Money:** Jan borrowed 10 Euros from Duncan and 20 Euros from Serge, but he has only 15 Euros left. How should Jan divide his 15 Euros?

No doubt, both Duncan and Serge *prefer* more money to less, so that envy-freeness recommends that Jan allots them equal amounts. Now, while Duncan and Serge may have the same (subjective) preferences, they have different (objective) *claims*: Duncan has a *claim* to 10, and Serge to 20 Euros, in virtue of the promise of Jan to give back the money. According to objective theories, fairness recommends the *proportional satisfaction* of claims so that, in Owing Money, fairness requires that Serge receive twice as much as Duncan. Although Owing Money is more naturally analysed along the lines of an objective theory of fairness, the division problems studied by welfare economics typically do not involve agents with claims, but with preferences. For these problems, then, criteria of fair division have to be formulated in terms of preferences.

Preference-based ‘no-envy’ theories of fairness have been very influential. Indeed, there are now various strands of fairness-related literatures in philosophy and economics in which different variants of the so-called

<sup>3</sup> The longstanding methodological debate about what kind of (if any) psychological content is implied by or should be attributed to the concept of *preference* can be safely left aside here.

<sup>4</sup> In philosophy, objective theories of fairness are mainly based on Broome (1990) who characterizes fairness as proportional satisfaction of claims, which is not a subjective notion (see also Rescher 2002). In economics, objective theories of fairness are put forward in the axiomatic and game-theoretic literature of fair division problems (reviewed in Thomson 2015). For a more comprehensive taxonomy and review of fairness theories in philosophy and economics, see Wintein and Heilmann (forthcoming).

‘no-envy’ principle play an important role, most prominently in: the literature in economics (Young 1994; Kolm 1996; Thomson 2011), including the algorithmic literature (Brams and Taylor 1996), various strands of normative economics (Moulin 2003; Fleurbaey 2008; Fleurbaey and Maniquet 2011), and political philosophy, including egalitarianism (Olson 2020; Fleurbaey 2021).

Given the importance of the ‘no-envy’ criterion for philosophy and economics alike, it is interesting that its modern<sup>5</sup> origins have been a matter of some ambiguity. Three attributions have become the norm in the literature: firstly, and most commonly, the ‘no-envy’ principle is attributed to Foley (1967). Secondly, references to either one of the two editions of Jan Tinbergen’s book *Redelijke Inkomensverdeling* (1946, 1953) are given. Most prominently, Thomson’s (2011, 395) extensive review notes that: “It is only in 1967 that an ordinal equity criterion designed for the evaluation of choices in concretely specified resource allocation models was first proposed: ‘no-envy’”.

However, in the same review, Thomson remarks in a footnote:

The idea had been formulated by at least one previous writer. Tinbergen (1953) devotes a few pages to a discussion of the no-envy test, explaining that he had developed it in conversations with the Dutch physicist Ehrenfest. However, it is thanks to Foley that the criterion has become known, and this author is usually credited with it. (Thomson 2011, 402n6)

Peyton Young, in his seminal 1994 book *Equity: In Theory and Practice*, also acknowledges that Tinbergen was the first to advance the modern ‘no-envy’ principle, when he writes that it was “first proposed in a very strong form by Tinbergen (1953)” (Young 1994, 11). There is also a third attribution, an anecdotal one, referring to conversations between Jan Tinbergen and his supervisor, Paul Ehrenfest, in the early 1920s. In an interview, published by Lambert (2012),<sup>6</sup> Serge-Christophe Kolm recounts the following:

Here is an interesting story. In 1924 in Holland there was a student of physics who was 18, a young man named Jan Tinbergen interested in justice. He asked his professor, Paul Ehrenfest, [...] ‘What do you think a just wage is?’ Ehrenfest thought a little and said ‘Well, if I

<sup>5</sup> Olson (2018) cites Denyer (2013) who discusses the earlier history of envy as a concept. For a comprehensive review of the various roles of envy in philosophy, including those beyond fairness, see also D’Arms (2017).

<sup>6</sup> We thank Kristi Olson for bringing this interview to our attention.

prefer my wage and the job I do, to your wage and the job you do, and conversely, this is kind of fair.’ Jan Tinbergen personally told me that story [...] at a conference in Paris in 1961. (Lambert 2012, 70)

And in his 1971 book *Justice et Équité*, Kolm ([1971] 1997) notes:

The criterion of Equity as no individual preferring any other’s allocation to her own, without any mention of equal liberty, has been introduced in economics by its mention by J. Tinbergen in application to equitable wages and occupations in *Redelijke Inkomensverdeling* (De Gulden Pers, Haarlem, 1946, in Dutch), following a suggestion from Tinbergen’s professor, the Dutch physicist Ehrenfest, in 1925. This previous suggestion by Ehrenfest was pointed out to me by Tinbergen in 1962 at a conference in Paris. (Kolm [1971] 1997, 6)

Naturally, the exact dates of this anecdotal attribution vary (for instance, Olson 2018 refers to 1921) and may be related to the fact that Tinbergen was a student of Ehrenfest from “around 1920” (Dekker 2021, 61) onwards.<sup>7</sup> Nevertheless, there is strong evidence for the fact *that* Ehrenfest and Tinbergen had substantial conversations about what is now known as the ‘no-envy’ criterion.<sup>8</sup>

In this article, we demonstrate that there is an additional, fourth, attribution: Tinbergen explicitly formulated the ‘no-envy’ principle in a 1930 article published in the Dutch journal *Mens en Maatschappij*, now translated into English in the *Erasmus Journal for Philosophy and Economics*.

### ***II.II. Tinbergen’s (1930) Formulation of the ‘No-Envy’ Principle***

The article “Mathematische Psychologie” starts by, and mainly consists of, presenting the essential ingredients of early welfare economics as developed by Edgeworth (in *Mathematical Psychics*, 1881) and Pareto. Following these two authors, Tinbergen introduces the methodology of analysing the consumption of agents that have ordinal preferences over bundles of goods by using *indifference curves*. He introduces the main uses of indifference curves and presents material up to what is today known as Edgeworth’s box.

He opens the introduction of the ‘no-envy’ criterion by commenting that he is convinced that the methods of Pareto and Edgeworth can also be fruitfully applied to problems in normative economics. Most prominently,

<sup>7</sup> For more detailed discussion about Ehrenfest and Tinbergen, see Boumans (1993) and Dekker (2021, chap. 4).

<sup>8</sup> See also the next section for textual evidence from Tinbergen’s ([1930] 2021) article itself.

they can be applied to the problem of determining a fair income distribution: “The mathematical-psychological method seems to be useful for the analysis of the common conception of justice with regard to the rules governing the distribution of economic goods” (Tinbergen [1930] 2021, 216). Directly after this passage, there is a long footnote, which contains textual evidence of the role of Paul Ehrenfest: “The following problems and solutions were developed in very animated discussions with Prof. Ehrenfest” (216n5).

Tinbergen acknowledges the importance of being precise in the definitions used for such an analysis, which he starts as follows:

The determination of ‘just distributions’, for example, the distribution of the total production among all individuals, must of course be preceded by a definition of justice. As far as I can see, the common conception of justice entails the equal treatment of equally situated individuals. (Tinbergen [1930] 2021, 216)

Tinbergen then goes on to discuss two types of situations: one in which the individuals in question are equal (already alluded to in the above quotation), and one in which there are differences between them. On the equality case, Tinbergen’s comments are straightforward:

A) Two identical workers perform the same amount of labour of a certain kind. Which distribution of the product can be considered just? The criterion mentioned above gives the solution of equal distribution: equal wages. (Tinbergen [1930] 2021, 216-217)

Next, Tinbergen raises the problem of differences between individuals, and introduces the ‘no-envy’ criterion as the solution:

However, it cannot provide a solution in problems involving individuals in *unequal* circumstances, as, for example, when one extends the above case A in one or more of the following ways:

- 1) the quantities of work are unequal;
- 2) the workers have different opportunity curves;\*
- 3) the type of work is different.

As soon as one of these cases arises, in which the circumstances (in the broadest sense) of the individuals are different, the possibility of applying the principle of equal treatment is no longer there. The question is, how to extend the criterion of just distribution to these cases. In my opinion, the requirement *that the in-*

*dividuals should not have the desire to occupy each other's position (in the broadest sense)* should be adopted as such. We postulate this answer to the above question without any further justification other than that the first-mentioned 'simple justice criterion' must be included as a special case, which is indeed the case. (Tinbergen [1930] 2021, 217; italics in the original)

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\* An open question for me here is whether it is sufficient for the psychological 'identity criterion' to hold for two people to be equal in terms of their ophelimity curves.

This passage is remarkable in a number of respects. Firstly, consider the careful introduction of three different dimensions on which differences between individuals might matter (we will return to this in the next section). Secondly, the footnote (included as footnote \* in our article) that clarifies the role of ophelimity<sup>9</sup> curves is remarkably careful as well. He seems to safeguard against taking them too literally: indifference curves are an important summary of an agent's psychology (and useful, as they can be elicited by experiment), but also have limitations in how much of the psychology of an agent they can capture.<sup>10</sup> Thirdly, and most importantly, Tinbergen presents the 'no-envy' criterion: 'individuals should not have the desire to occupy each other's position (in the broadest sense)'. Fourthly, he mentions how equality can be subsumed under it as a special case.

Tinbergen then goes on to illustrate how his 'no-envy' criterion can be applied. For this he considers two agents, who work for  $x_1$  and  $x_2$  hours respectively. They have (ordinal) preferences over pairs ( $x$  = hours worked,  $z$  = wage received) represented by two functions,  $w_1$  and  $w_2$ . The simplest example that Tinbergen discusses is when the two agents have the same preferences, that is when  $w_1 = w_2 = w$ . Then, the 'no-envy' principle dictates that:

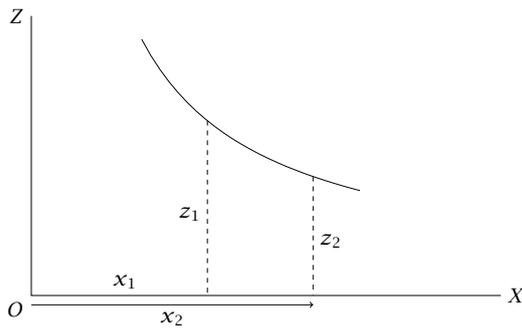
$$w(x_2, z_2) \leq w(x_1, z_1) \quad w(x_1, z_1) \leq w(x_2, z_2) \quad (1)$$

Combining the 'no-envy' conditions for both agents for this case, we see that  $w(x_2, z_2) = w(x_1, z_1)$ . Hence, if we know the hours  $x_1$  worked and wage  $z_1$  received by agent 1, we can find out which combinations of work-

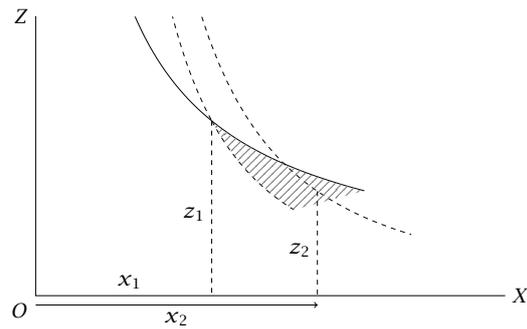
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<sup>9</sup> Tinbergen uses the Paretian term 'ophelimity'—we use the current terminology of indifference curves in this article.

<sup>10</sup> Whether Tinbergen also had in mind the idea that the presence of feelings of envy and the preference-based 'no-envy' criterion do not necessarily imply each other (discussed in section II.I) is not entirely clear from the passage and footnote. It is certainly consistent with it.



**Figure 1:** ‘No-envy’ (Tinbergen [1930] 2021, Figure 7).



**Figure 2:** ‘No-envy’ with different indifference curves (Tinbergen [1930] 2021, Figure 8).

ing hours and wage for agent 2 would be fair and would result in ‘no-envy’. Tinbergen ([1930] 2021) explains, referring to Figure 1, that ‘no-envy’ requires that agent 2’s combination  $(x_2, z_2)$  has to be located on the same indifference curve as agent 1’s  $(x_1, z_1)$ .

For different indifference curves, the ‘no-envy’ criterion reads:

$$w_1(x_2, z_2) \leq w_1(x_1, z_1) \quad w_2(x_1, z_1) \leq w_2(x_2, z_2) \quad (2)$$

We discuss equation (2) by referring to Figure 2.

Figure 2 displays two points,  $(x_1, z_1)$  and  $(x_2, z_2)$ , for which (2) is satisfied. In order to satisfy (2), the point  $(x_2, z_2)$  must be under the indifference curve of agent 1 (continuous line) through  $(x_1, z_1)$  while the point  $(x_1, z_1)$  must be under the indifference curve of agent 2 (dashed line) through  $(x_2, z_2)$ . Indeed, these conditions are fulfilled for the points,  $(x_1, z_1)$  and  $(x_2, z_2)$ , that are displayed in Figure 2. In contrast to the case discussed above, in which the two agents have identical indifference curves, a particular combination  $(x_1, z_1)$  of (hours worked, wage paid) for agent 1 does not uniquely determine the combination  $(x_2, z_2)$  for agent 2 that results in envy-freeness. Indeed, with  $(x_1, z_1)$  given as in Figure 2, the grey area marks possibilities for  $(x_2, z_2)$  that result in envy-freeness.

This concludes the presentation of Tinbergen’s early 1930 formulation of the ‘no-envy’ criterion. Its remarkably careful presentation as well as concrete exposition via the example strikes us as a central finding in two ways. One, it further clarifies the somewhat ambiguous origins of the modern ‘no-envy’ criterion. Two, by combining a very general formulation (that mentions social positions ‘in the broadest possible sense’) with a concrete exposition of application (two agents with different wages and

hours worked), he anticipates different types of applications of it. We now turn to briefly introducing Tinbergen's (1946, 1953) formulation.

### ***II.III. 'No-Envy' in Tinbergen (1946, 1953)***

Tinbergen's discussion in his book *Redelijke Inkomensverdeling* does not contain comments on the theoretical aspects of welfare economics that were present in his 1930 article. However, with respect to 'no-envy', it is both conceptually and empirically richer than the 1930 article, containing a number of reflections on the relation between 'no-envy' and welfare. Partly, he gives broader context to his sparse discussion in the 1930 article, and partly, he also motivates some of the underlying ideas further. In particular, the three dimensions of inequality (quantity of work, preferences, type of work) mentioned in the 1930 article are discussed more thoroughly.

Tinbergen starts his discussion on 'no-envy' in the book by noting that a two-fold question should be answered: how to *produce* the largest amount of welfare, and how to *divide* the benefits and burdens associated with this production fairly (Tinbergen 1953, 51)? As the demands of production and the demands of fairness are typically not aligned, one needs to find a harmonious synthesis between them, for which Tinbergen reserves the term *redelijkheid*, which could be translated as "reasonableness". It figures in the title of the book (*Redelijke Inkomensverdeling*, which could be translated as "Reasonable Income Distribution"). Reasonableness refers to the synthesis between the demands of production and fairness associated with income distributions.

After re-affirming the *equal treatment of equals* principle for contexts in which individuals are similar, Tinbergen laments the fact that principles guiding dissimilar conditions are not available. Tinbergen (1953, 53) notes that the same amount (of income) for all will not do: rewarding easy, low-skilled work and hard, high-skilled work equally conflicts with our sense of fairness. Tinbergen suggests that this conflict can be explained by the principle that fairness requires that everyone is equally *happy*: if one puts in more effort, works hard, or is ill, fairness requires that one is compensated so that, after compensation, all are equally happy. Should we then say that fairness requires the *equal happiness of all*? Tinbergen argues that *equal happiness of all*, albeit conceptually attractive, is not tenable as a principle that specifies the demands of fairness:

This now seems a very clear description of our goal with respect to income distribution: an equal amount of happiness for all. On closer inspection, however, it is unusable for offering practical guidance. For

how to determine whether two people are equally happy? (Tinbergen 1953, 54; translation by the authors)<sup>11</sup>

Tinbergen argues that we cannot determine whether two people are equally happy, which dismisses *equal happiness for all* as a principle that specifies the demands of fairness. In modern terminology, we would say that Tinbergen dismisses the *equal happiness for all* principle because it relies on the interpersonal comparison of utility which, according to Tinbergen (1946, 1953), is non-sensical. But then, if one abstains from such interpersonal comparisons, how are we to formulate the demands of *fairness*?

Tinbergen again introduces the ‘no-envy’ criterion as an answer to precisely this question. The main idea is that each individual evaluates every other individual’s position in terms of their *own* preferences, so that interpersonal comparisons are avoided:

We say that two people have been treated justly with respect to each other *when neither of them wants to trade places with the other*. By ‘place’ we mean, figuratively, all circumstances: in particular the profession and the income, but also the personal circumstances, such as health, wellbeing and health of the family, degree of development, ability, etc. (Tinbergen 1953, 55; translation by the authors)<sup>12</sup>

Tinbergen goes on to discuss this principle in relation to income distribution, and links it to questions of how to deal with, on the one hand, differences in innate abilities and health and, on the other hand, differences in effort, desert, responsibility, motivation, and incentive. Both might present problems in terms of equalizing. Regarding the first set of issues, it may be wholly impossible to create a state of the world in which innate abilities and health are distributed such that no one wants to trade places (this point is already mentioned in Tinbergen [1930] 2021, section V). Regarding the second set of issues, it may not only be impossible but also undesirable to create a state of the world in which aspects such as effort, desert, responsibility, motivation, and incentive are distributed such that

<sup>11</sup> The original in Dutch: “Dit lijkt nu een heel duidelijke omschrijving van ons doel inzake de inkomensverdeling: een even groot geluk voor ieder. Bij nader inzien is hij echter toch onbruikbaar, om als praktisch richtsnoer te dienen. Want hoe moeten wij uitmaken, of twee mensen even gelukkig zijn?” (Tinbergen 1953, 54).

<sup>12</sup> The original in Dutch: “Wij zeggen, dat twee personen rechtvaardig t.o.v. elkaar zijn behandeld, *wanneer zij geen van beiden met de andere van ‘plaats’ willen ruilen*. Met ‘plaats’ bedoelen we dan overdrachtelijk alle levensomstandigheden: in het bijzonder het beroep en het inkomen, maar daarnaast ook de persoonlijke omstandigheden, zoals gezondheid, gootte en gezondheid van het gezin, grad van ontwikkeling, bekwaamheid, enz” (Tinbergen 1953, 55; italics in the original).

no one wants to trade places. In relation to this, Tinbergen stresses that the principle of ‘no-envy’ is a *pro tanto* principle: it is but one criterion in judging social states of affairs. And so, for Tinbergen, the issue of a ‘reasonable’ income distribution is to seek balance between considerations of efficient production and fair division of welfare.

There is thus a slightly expanded discussion of the ‘no-envy’ criterion in the 1946/1953 book when compared to the 1930 article. Moreover, in the 1930 article, he gives a concrete example of two individuals to illustrate the principle (presented in section II.II), whereas in his book he repeatedly insists that the ‘no-envy’ criterion applies to *social groups*: “To avoid misunderstanding we repeat once more that [the ‘no-envy’ criterion] albeit explained via examples involving individual persons, is meant to apply to social groups” (Tinbergen 1953, 59).

Otherwise, both formulations of the criterion itself are non-committal and open about the domain of application. For instance, Tinbergen ([1930] 2021, 217) references social positions “in the broadest sense” and the above quotation from the 1946/1953 book refers to ‘figuratively, all circumstances’. It is helpful to keep these slight ambiguities in mind when we discuss other early contributions in the ‘no-envy’ literature.

### III. THE ‘NO-ENVY’ CRITERION AFTER TINBERGEN

#### *III.I. Comparing Different Variants of the ‘No-Envy’ Criterion*

The 1930 and 1946/1953 formulations of the ‘no-envy’ criterion by Tinbergen raise the question of how these compare to the other early proposals of it. We think it is useful to consider two dimensions of comparison.

Firstly, applying the ‘no-envy’ criterion can differ in terms of the alternatives over which the preferences are considered, that is, the ‘objects of envy’, so to speak. It is interesting to note that this is by and large an interpretative issue—one can always formulate the ‘no-envy’ criterion in terms of an allocation of *something*. These might be goods with a varying degree of complexity, or, as Tinbergen’s 1930 example has it, a combination of hours worked and wages received. In Tinbergen’s richer 1946/1953 discussion, there are some hints that he thinks of the allocation primarily in terms of income (also referenced in the title of his book), which he saw as being tied to societal ‘places’ or ‘positions’. Regarding the latter, however, it is important that he already mentions social positions in the 1930 article, as discussed above. So, already in Tinbergen’s early article (as well as in the book), there are different possible ‘objects of envy’ mentioned.

A second dimension on which ‘no-envy’ criteria may differ is the ecology of concepts within which they are conceived of and applied to. Simply

put, what are other theoretical concepts that the ‘no-envy’ criterion is used in conjunction with? Tinbergen’s sparse 1930 example makes no mention of any other concept (he even avoids explicitly mentioning utility).<sup>13</sup> However, his 1946/1953 book conjures many other concepts, including efficiency of production, but also innate ability, endowments in relation to the social position of the individuals concerned, and effort. But he is not, in contrast to later work in economic theory, concerned with the efficiency of *distribution* yet.

There are, no doubt, more dimensions on which different formulations of the ‘no-envy’ criterion might be compared. However, we are not aiming at a complete taxonomy of ‘no-envy’ criteria in this article that addresses all these issues. Rather, we want to stay focused on presenting and contextualising Tinbergen’s 1930 contribution. Arnsperger (1994, 157) mentions the following as “seminal references” for the ‘no-envy’ criterion: Tinbergen (1946), Foley (1967), Schmeidler and Yaari (1971), Kolm ([1971] 1997), and Varian (1975). We take Foley (1967) as the main source for comparison. We do so in part because he has been the main historical reference in the ‘no-envy’ literature. However, we will also see that the differences in the intended domain of application in Tinbergen (1930) and Tinbergen (1946, 1953) connect very well to issues in the interpretation of ‘no-envy’ within Foley (1967).

### ***III.II. Foley (1967) and Tinbergen (1930, 1946/1953)***

Foley (1967) has hitherto been considered the *locus classicus* for the ‘no-envy’ criterion in the literature. He introduces it as follows.

There is an intuitive notion of inequality. If one person consumes more of every good (including leisure) than another, he is better off. If two people have identical preferences and one is in a position preferred by both, they cannot have equal welfare. This suggests a new way to define equality even when preferences are diverse: an allocation is equitable if and only if each person in the society prefers his consumption bundle to the consumption bundle of every other person in the society. (Foley 1967, 74)

It seems straightforward that the underlying idea is exactly the same as in Tinbergen (1930, 1946/1953). Interestingly, considering the above passage alone might suggest that Foley is interested in a narrow object of envy: that is, comparing preferences between consumption bundles. It

<sup>13</sup> Observe, however, that Tinbergen uses the letter *w* in his 1930 article in the preference comparison. This might be taken as a hint at a *welfarist* interpretation.

seems to us that when viewed in isolation like this, it is easy to reduce Foley's 'no-envy' criterion to such a quite narrow version. To see this, we discuss a typical example that is used in the literature to introduce Foley's formulation of 'no-envy'. Consider an economy with two goods, *food* and *clothes*, and two agents, *Alice* and *Bob*, whose (ordinal) preferences for the goods are represented by the following utility functions:

$$U_A(f, c) = 2f + c \quad U_B(f, c) = f + 2c$$

Suppose that there are 4 units of *food* and 4 units of *clothes* in total. An *allocation*  $(x_A, x_B)$  distributes the total endowment amongst *Alice*, who receives  $x_A = (f_A, c_A)$ , and *Bob*, who receives  $x_B = (f_B, c_B)$ . An allocation for this economy is envy-free just in case it satisfies equation (3):

$$U_A(f_B, c_B) \leq U_A(f_A, c_A) \quad \text{and} \quad U_B(f_A, c_A) \leq U_B(f_B, c_B) \quad (3)$$

Clearly then, Foley's notion of 'no-envy' if articulated by (3) is formally equivalent to Tinbergen's 'no-envy' notion as articulated by (2). However, the interpretation of the objects of comparison is slightly different: Tinbergen (1930) introduced preferences over pairs of wages earned and hours worked, whereas the above formulation concerns the allocation of consumption bundles.

Was Foley more concretely minded than Tinbergen in the domain of application of the 'no-envy' criterion? Young (1994) suggests that he was. He writes that Tinbergen puts forward the 'no-envy' principle "in a very strong form" (Young 1994, 11). He takes issue with Tinbergen specifying 'no-envy' in terms of agents not wanting to trade *places* or 'positions in the broadest sense'. For, as the notion of a *place* is very general and may involve non-transferable personal characteristics, such as height, health, intelligence, or age, Tinbergen's 'no-envy' principle may be impossible to realize. Young illustrates this non-realizability via an example where person *A* envies person *B* for being tall, whereas *B* is indifferent between being tall or short. Then, a monetary compensation of *A*'s envy of *B*'s height will result in *B* being envious of *A*'s money. Hence, there is envy come what may, so that an envy-free arrangement does not exist. According to Young, this example illustrates that "there may be no reasonable way of eliminating envy in Tinbergen's sense", and he then quickly turns to "the more pragmatic formulation" (Young 1994, 11) due to Foley (1967) which, according to Young, defines envy-freeness not in terms of societal *places* but, more narrowly, only in relation to particular allocations of goods.

It is true that Tinbergen (1946, 1953) applies the ‘no-envy’ criterion to ‘social groups’. However, he discusses in quite some detail that people’s inability to move freely between associated social ‘places’ presents serious problems for realizing fair, envy-free income distributions. But also, Tinbergen (1930) contains concrete, and in that sense pragmatic, applications of the ‘no-envy’ criterion that we discussed in section II.I. Indeed, in the models discussed in Tinbergen (1930), the non-existence of envy-free allocations is not at stake.

The differences between Tinbergen and Foley should not be overemphasized. For Foley—even though one may suitably make his proposal concrete in the way introduced above—was not necessarily that much ‘more pragmatic’ than Tinbergen, as Young (1994) suggested. Foley explains some issues of interpretation of the consumption bundles in question, which we quote here at length. It strikes us as interesting in terms of the interpretation and intended domain of application of the ‘no-envy’ criterion:

Ask each person to imagine changing places with every other, not by exchanging incomes, but by experiencing the material aspects of that person’s life. If no one is willing to change, the allocation is equitable. The farmer might be willing to take on the bricklayer’s apartment, TV, dinners, steady working hours, and Saturdays in a bar in exchange for his own homestead, home-grown food, and freedom from supervision if the city air was clean but not if it was dirty. In applying the test there are some simple qualifications. First, the consumption patterns compared must be lifetime plans to wash out differences of time preference and other noncomparabilities. The postponement by professional men of marriage and of even modest incomes must be taken into account when the salesman views their high rates of consumption later in life. Second, the comparison can be pushed only so far, and some common sense procedures are necessary when the comparison involves orchestra conductors, painters, chess masters, and so on. The difficulty here is that if a gas station attendant has the desire to be a painter but not the ability, it may be necessary to make the painter’s life very unattractive in other ways before the gas station attendant will prefer his own; so unattractive, perhaps, that the painter will envy the attendant while the attendant is still envying him. These cases must be interpreted flexibly; either equivalents to the talents must be postulated which the gas station attendant does possess, or reasonable alternatives framed that abstract from the glamour and prestige of certain activities. (Foley 1967, 75)

In other words, it seems that Foley (1967) is—quite similarly to Tinbergen (in both 1930 and 1946/1953)—also aware of the fact that a too narrow interpretation of what is compared in the preference used for the ‘no-envy’ criterion may not be opportune. In relation to this, it is also important to note that Foley (1967) did not himself give the concrete example that we used above to illustrate his view. In contrast, Tinbergen (1930) did provide two concrete cases of specific hours worked and wages earned in order to illustrate the ‘no-envy’ principle.

It is perhaps not desirable to get too bogged down in discerning the precise meaning that Tinbergen and Foley were trying to convey in what were not fully worked-out theoretical developments, but rather statements of an ethical principle that seemed plausible to them. What seems important to us is that both authors are aware that there are different kinds of possible objects of envy (ranging from concretely specified consumption bundles to social states in the broadest sense). This issue seems particularly salient as the literature on fair allocation in economic theory has developed addressing the envy-free (and Pareto-optimal) allocation of goods, whereas Tinbergen’s key concern has remained a fair, or reasonable, income distribution.<sup>14</sup>

### ***III.III. ‘No-Envy’ after Tinbergen and Foley***

Varian (1975) expanded on Foley (1967) by proving results on the relation between fairness and efficiency. This marks a clear shift in the literature. Whereas Tinbergen ([1930] 2021) and Foley (1967) mainly stated the ‘no-envy’ principle, and included some thoughts on the different types of objects of envy, the ecology of concepts they considered alongside ‘no-envy’ did not yet play a major role for them. As mentioned in section II.III, Tinbergen (1946, 1953) discussed the demands of production efficiency alongside the fair distribution of welfare. However, Tinbergen also refrained from offering a theoretical discussion of their relation along the lines of Varian (1975) and other authors after the latter.

In order to appreciate Varian’s (1975) contribution, we continue the above example of an economy with two goods, *f* food and *c* clothes, and two agents, *Alice* and *Bob*, whose (ordinal) preferences for the goods are represented by the following utility functions:

$$U_A(f, c) = 2f + c \qquad U_B(f, c) = f + 2c$$

<sup>14</sup> For an elaborate discussion on the latter point, see Dekker (2021, 375–392).

As before, suppose that there are 4 units of *food* and 4 units of *clothes* in total. Now, quite a few envy-free allocations for the example at hand exist. In particular, allocation  $a = ((3, 0), (1, 4))$  is envy-free: Alice receives  $(3, 0)$  which gives her a utility of 6 so that she does not envy Bob's  $(1, 4)$  which would also give her a utility of 6. Likewise, Bob's  $(1, 4)$  yields him a utility of 9 so that he does not envy Alice's  $(3, 0)$  which would only give him a utility of 3. Not only is  $a$  envy-free, but it is also (Pareto)-*efficient*—that is, there is no allocation  $x$  which is strictly preferred to  $a$  by one individual while  $x$  is weakly preferred to  $a$  by all other individuals. The notions of efficiency and envy-freeness do not always coincide, as they do in allocation  $a$ . For instance,  $((2, 2), (2, 2))$  is envy-free but not efficient, whereas  $((0, 0), (4, 4))$  is efficient but not envy-free.

From Varian's (1975) contribution onwards, a large literature developed in economic theory that explored the relations between efficiency (via the Pareto principle) and fairness (via the 'no-envy' criterion), which is reviewed in Arnsperger (1994), Kolm (1996), and Thomson (2011). Fleurbaey (2008) makes seminal contributions at the intersection of philosophy and economics, and Olson (2018) and Fleurbaey (2021, section 6) review philosophical contributions.

The strand of literature on envy-freeness succeeding Varian (1975) is by and large concerned with *existence* results: it is concerned with the question under what conditions envy-free allocations may or may not *exist*. The question of how rational, self-interested individuals may *end up* in an envy-free allocation, that is by bargaining, trading or, more generally by 'following the rules of the games', is not discussed. However, in the literature on 'algorithms for fair division' that addresses these very questions, the notion of 'no-envy' plays a prominent role, as acknowledged by two of its most prominent contributors:

Our approach to fair division is distinctive not only in combining properties, algorithms, and applications but also in elevating the property of 'envy-freeness', and procedures that generate envy-free allocations, to a central place in the study of fair division. Roughly speaking, an envy-free division is one in which every person thinks he or she received the largest or most valuable portion of something—based on his or her own valuation—and hence does not envy anyone else. Although the concept of envy-freeness has been used in the mathematics literature on fair division for almost 40 years (Gamow and Stern, 1958) and in the economics literature for almost thirty years (Foley, 1967), only recently have several algorithms been developed that guarantee

envy-freeness in a wide variety of situations. (Brams and Taylor 1996, 2)

As demonstrated in this article, the concept of envy-freeness has been used in the economics literature for more than 90 years now. And, as mentioned in the above quotation, the ‘no-envy’ criterion is indeed applicable to a wide variety of problems. In this analysis, envy-freeness plays an important role. However, as this literature assumes that preferences are interpersonally comparable, further, non-ordinal fairness criteria become available and are studied. To illustrate these non-ordinal criteria, we briefly consider the most well-known problem studied in the algorithms for fair division literature: the so-called ‘cake-cutting problem’.

A single rectangular cake has to be divided, in two pieces that result from a single vertical cut, amongst Alice and Bob. The cake need not be homogeneous—that is, it may be topped with chocolate on the left but with strawberries on the right—and Alice and Bob need not have the same preferences<sup>15</sup> for chocolate and strawberry. What properties should a division  $(A, B)$  of the cake, allotting piece  $A$  to Alice and piece  $B$  to Bob, satisfy? The following two familiar properties are attractive:

**Envy-freeness:**  $U_A(B) \leq U_A(A)$  and  $U_B(A) \leq U_B(B)$

**Efficiency:** For no  $(A', B')$ :  $U_A(A) \leq U_A(A')$  and  $U_B(B) < U_B(B')$  or  $U_A(A) < U_A(A')$  and  $U_B(B) \leq U_B(B')$

Envy-freeness and efficiency only require ordinal utilities which need not be interpersonally comparable. Cake divisions may also satisfy two further attractive properties, which are more demanding, concerning the comparability of utilities. *Proportionality* requires that both Alice and Bob receive, according to their own valuation, at least half of the cake’s value, whereas *equitability* requires that Alice and Bob receive the same utility from their cake pieces:

**Proportionality:**  $U_A(A) \geq (1/2)U_A(\text{cake}) = 1/2$  and  
 $U_B(B) \geq (1/2)U_B(\text{cake}) = 1/2$

**Equitability:**  $U_A(A) = U_B(B)$

<sup>15</sup> The cake preferences of Alice and Bob are assumed to be expressed by additive and normalized utility functions  $U_A$  and  $U_B$ . Additive means that for any two pieces of cake  $X$  and  $Y$ ,  $u_i(X \cup Y) = u_i(X) + u_i(Y) - u_i(X \cap Y)$ . Normalized means that receiving nothing yields a utility of 0 whereas receiving the whole cake yields a utility of 1.

Arguably, envy-freeness, efficiency, proportionality, and equitability are desirable properties for an allocation to have. Moreover, one may even argue that, for problems with interpersonally comparable preferences, *fairness* requires more than just envy-freeness and also, say, proportionality or equitability.

However, what really sets apart the literature on algorithms for fair division is not the specification of non-ordinal ‘fairness criteria’, but the focus on *algorithms*. To illustrate, suppose that Alice and Bob have to divide the cake but there are no parents, government, or impartial observer to do the cutting and dividing for them, what should they do? They could, for instance, use the *cut-and-choose* algorithm which, intuitively, seems to be a fair way of solving the problem. According to *cut-and-choose*, one individual, say Alice, cuts the cake after which Bob is allowed to choose a piece, leaving the other piece for Alice. *Cut-and-choose* is intuitively fair and it can be shown that, when Alice and Bob cut and choose rationally, it will result in an allocation that is envy-free and proportional, but not necessarily equitable. Further, the resulting allocation is efficient with respect to the class of all single-cut allocations.

As discussed in sections II.II and II.III, Tinbergen did not consider preferences to be interpersonally comparable. Moreover, he did not consider or discuss algorithms in the sense of the literature discussed above. Indeed, Tinbergen ([1930] 2021, 218–219) assumes that when an envy-free allocation exists, it will be reached “automatically”. However, Tinbergen was concerned with conditions under which envy-free allocations could be reached. Indeed, he writes that an envy-free allocation exists under certain conditions: “When any individual can indeed, if so desired, reach a state (in the broadest sense) equal to that of any other individual: when there is ‘free mobility’ in all respects” (Tinbergen [1930] 2021, 219).

He then notes that this free mobility may be absent in a wide variety of circumstances, due to a wide variety of causes and, in the last section of Tinbergen ([1930] 2021), presents various examples of this absence. This vindicates our earlier remark, made in section III.II, that Tinbergen was well aware of the difficulties surrounding the concept of social states in the broadest sense.

#### **IV. TINBERGEN AND CONCEPTUAL DISCUSSIONS OF FAIRNESS**

We now turn to two general observations about the relation between Tinbergen’s contributions and conceptual discussions of fairness.

Firstly, in our translation of Tinbergen’s 1930 article (Tinbergen [1930] 2021), we have stayed as closely as possible to the original, and therefore

chosen to use the words ‘just’, ‘justly’, or ‘justice’ where one might also have used the words ‘fair’, ‘fairly’, or ‘fairness’. It is worth mentioning that the usage of these words does not follow consistent rules in the contemporary literature in philosophy and economics. Furthermore, there are also a number of other closely related labels that are used in the various strands of literatures, and the usage of ‘justice’ and ‘fairness’ varies. One trend in terminology that seems to prevail, though, is that ‘fairness’ is more often used for smaller and more easily delineated contexts, whereas ‘justice’ is often used for broader, more encompassing problems of larger scale. Peyton Young comments:

Fairness in [a] global sense is concerned with the proper distribution of resources, rights, duties, opportunities, and obligations in society at large. This grand theme has animated political philosophers since antiquity, from Plato’s and Aristotle’s conceptions of the ideal state, to the social contract theories of Hobbes, Locke, and Rousseau, to the more modern theories of Rawls, Nozick and Walzer. I shall refer to these as theories of *social justice*. (Young 1994, xi)

We agree with Young that it is useful to distinguish between ‘local’ and ‘global’ fairness problems, and also with his assertion that problems of a local scale can be solved without invoking ‘global’ theories (whether one wants to refer to them as concerned with ‘fairness’ or ‘justice’).<sup>16</sup> Here, it is interesting that the ‘no-envy’ criterion might be applied to contexts of different scale. Indeed, this issue is linked to the different suggestions for the object of envy in the texts by Tinbergen and Foley, which we analyzed in sections II and III. Simply put, one might use the ‘no-envy’ criterion in local settings (such as cake cutting, or in general the division of a very simple bundle of goods), or one might conceive of it globally, in terms of judging the fairness of social structures (such as when conceiving of whole life histories of individuals that touch on all kinds of social arrangements).

Secondly, we comment on a structural coherence between Tinbergen’s starting point in theorizing about fairness and that of key fairness theories in philosophy and beyond. There is broad agreement between fairness theorists in both philosophy and economics that fairness requires (1) ‘the equal treatment of equals’. However, a fairness theory also needs to specify what fairness requires for (2) ‘the treatment of unequals’. Tinbergen also comments on the different status of (1) and (2), both in his 1946/1953 book and in his 1930 article:

<sup>16</sup> For a more detailed discussion, see Young (1994) and Wintein and Heilmann (forthcoming).

However, one principle is generally accepted; one could call it ‘same monks, same robes’. Those who are in similar condition should also be treated equally. There is no equally clear principle about dissimilar conditions. (Tinbergen 1953, 50; translated by the authors)<sup>17</sup>

As far as I can see, the common conception of justice entails the equal treatment of equally situated individuals. [...] However, it cannot provide a solution in problems involving individuals in *unequal* circumstances. (Tinbergen [1930] 2021, 216–217)

Tinbergen’s ([1930] 2021) ‘no-envy’ principle is thus a solution for (2) which, as he remarks, includes (1) as a special case.

So, Tinbergen’s ‘no-envy’ principle substantiates (1) and (2) for cases where individuals have ordinal preferences. But, more generally, there is broad consensus on the fundamental two-fold fairness concern in the literature on fairness at large, such as in Aristotle’s *Nichomachean Ethics*. The two-fold fairness concern is also visible in the algorithmic literature cited in section III.III, via the principles equitability and proportionality for interpersonally comparable utilities.

Indeed, the two-fold fairness concern also extends across the divide between subjective and objective fairness theories. As remarked in section II.I of this article, there are theories that view fairness as something subjective (such as those theories based on a ‘no-envy’ criterion) and there are also theories that conceptualize fairness as objective. Now, objective fairness theories also strive to give meaning to the two-fold fairness concern of (1) equal treatment of equals and (2) unequal treatment of unequals. Consider Broome’s (1990) influential account of objective fairness, according to which fairness requires to ‘satisfy claims in proportion to their strength’. This principle clearly gives substance to (2); and it includes (1) as a special case, as it follows that claims with equal strength require equal satisfaction.

We find it remarkable that both the distinction between local and global fairness as well as the two-fold fairness concern are explicitly visible in Tinbergen’s early contributions. We also think it shows that, more generally, conceptual discussions of fairness can fruitfully be viewed as competing specifications of such general desiderata of fairness theories. We develop more specific discussions of this theme elsewhere (Heilmann and Wintein 2017; Wintein and Heilmann 2018, 2020, forthcoming). Here,

<sup>17</sup> The original in Dutch: “Een principe is wel algemeen aanvaard; men zou het door de uitspraak: ‘gelijke monniken, gelijke kappen’ kunnen aanduiden. Wie in hetzelfde geval verkeren, moeten ook gelijk behandeld worden. Over ongelijke gevallen bestaat echter geen duidelijk principe” (Tinbergen 1953, 50).

we stress the intriguing parallels between Tinbergen's and many other contributions in the interdisciplinary discussions about fairness in philosophy, economics, and beyond.

## V. CONCLUSION

We have revealed that Jan Tinbergen introduced 'no-envy' as a fairness criterion in his article "Mathematische Psychologie" published in 1930 in the Dutch journal *Mens en Maatschappij* and translated as "Mathematical Psychology" in 2021 in the *Erasmus Journal for Philosophy and Economics*. This fact calls for a revision in the attribution of the 'no-envy' criterion in the fairness related literature.

We discussed Tinbergen's (1930, 1946/1953) and Foley's (1967) formulation of the 'no-envy' criterion. We found that there are similarities: both authors speak of the criterion being applied to an individual's 'place' or 'position' within society, but also slight differences in the interpretation of the object of envy: Tinbergen (1930) considered hours worked and wages earned, Tinbergen (1953) considered income, and Foley (1967) considered consumption bundles. We also briefly commented on how Tinbergen's notion of 'no-envy' connects to that used by the more recent literature on 'no-envy' and, more broadly, fairness discussions in general.

Jan Tinbergen's contributions to theories of redistribution are well known—here we have shown that he has made crucial, and early, contributions to the conceptual discussion of fairness as well.

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