

**Review of Richard Pettigrew's *Dutch Book Arguments*.
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I. INTRODUCTION AND SUMMARY

A central notion of decision theory is that of credence: the idea that beliefs come in degrees. I could be 80% confident that there is a sheep standing in the field before me and 20% confident that it is, instead, a dog disguised as a sheep. Not all credences make sense, however. My credence that, in this field, there is a sheep hidden behind a disguised dog cannot be greater than my credence that there is at least one disguised dog. For the former cannot be true unless the latter is as well. To understand how credences relate, a rich academic literature has proposed several laws of credence: these are laws that rational persons should abide by when forming credences. More recently, some have sought to establish such laws with stronger formal foundations. Richard Pettigrew's *Dutch Book Arguments* constitutes a remarkable attempt to define and refine laws of credence using Dutch Books—that is, betting situations in which the gambler loses money for sure. By analysing these situations, this book skilfully progresses towards the formal establishment of some laws of credence. It thereby contributes in many ways to a larger effort aimed at ascertaining the cogency of formal arguments for decision theory (see Vineberg 2022).

Dutch Books can be useful formal tools in the following manner. A Dutch Book refers to (a series of) bets that, once accepted, ensure that a gambler loses money. Still, a gambler's credences can be so that she must or might nevertheless accept the Dutch Book. She is thus guaranteed to lose money. Since her loss is due to the state of her credences, they appear faulty. As such, Dutch Books can reveal different ways in which one's credences are irrational, from a formal perspective. Several laws governing these credences can then be defined so that, if violated, the gambler will incur a loss.

Establishing the intuitiveness of these laws is the first challenge that the book takes up. Following a brief overview, the second section discusses five laws of credence. For each, the reader follows a protagonist

who accepts losing bets. Take Norma, a climate scientist, who has to estimate the likelihood of three possible worlds: the average temperature increase in one hundred years will either be *Low* (at most 0°C), *Medium* (between 0°C and 1°C), or *High* (1°C and above). Surprisingly, the sum of her credences in either *Low*, *Medium*, or *High* does not equal 100%, but only 90%. Since all possible worlds are considered, why is Norma not fully confident that one of them will obtain? Such credences seem irrational. With clarity and simplicity, Pettigrew shows that they are indeed irrational: there exists a Dutch Book against Norma that she must accept even though it results in her losing money. As such, he introduces the first law of credence, *Normalisation*, according to which one's credence in a necessarily true proposition should be maximal (that is, equal to 100%). Had Norma abided by *Normalisation*, she would not have been susceptible to a Dutch Book loss.

Similarly, Pettigrew adds protagonists to propose four other laws of credence. Finella breaks *Finite Additivity*: her credences in *Low* and *Medium* are 30% and 40% respectively, so her credence in both should be the sum of her credences in each (i.e., 70%), but they are not. Constanze breaks *Countable Additivity*: her mistake is akin to Finella's, except that her credences concern infinitely many propositions. Reg fails to observe *Regularity*: he cannot rule out the possibility that the average temperature increase will be *High*, so his credence in this possible world should be positive, yet it equals zero. Finally, Pritpal breaks *Principal Principle*: he has a positive credence that the objective chance that a storm reaches the United Kingdom by midnight is 50%, yet his credence in this proposition conditional on its objective chance is not equal to 50%, but it should be.

While these laws are first intuited, the third section refines them until a more rigorous formulation of each is reached. Only one (*Countable Additivity*) cannot be worked out, and so it is abandoned. Moreover, *Normalisation* and *Finite Additivity* are considered together as *Probabilism*. Thus, the section culminates in three final formulations of Dutch Book arguments (33–34). While the arguments are elaborate, their common structure can be summarised in this way:

(DB1) Suppose that a gambler's credences require or permit her to accept certain bets; then,

(DB2) If and only if her credences violate one of the three laws, they can be exploited to make her lose money;

(DB3) If her credences are exploitable (and there exists no alternative, unexploitable credences), then her credences are irrational.

The fourth section offers a pleasant *caesura* during which the reader is introduced to diachronic laws of credence. What happens when a gambler updates her credences after she acquires new evidence? Are there Dutch Books in such situations too? Pettigrew suggests that anyone who updates their credences in light of new evidence could be subject to a Dutch Book. Nevertheless, he refrains from concluding that updating is irrational. Rather, he recommends focusing not on actual updating behaviour but on the updating *rule*, as this move enables to make a more fine-grained distinction between rational and irrational updating.

The fifth section resumes the discussion where the third section left it. It addresses two challenges raised against the first premise (DB1) of the final Dutch Book arguments. The first one is due to Hedden (2013) who argues that, when credences are non-probabilistic, the betting behaviour prescribed under DB1 is no longer aligned with that of expected utility theory. In such cases, DB1 may arguably provide a flawed account of rationality. Pettigrew denies that expected utility should be the yardstick against which other accounts, such as DB1, are evaluated. DB1 could offer a plausible account of decision theory as well. Moreover, with non-probabilistic credences, expected utility might in fact lead to making suboptimal decisions, unlike DB1. Interestingly, Pettigrew goes on to elaborate a disjunctive norm combining DB1 and expected utility theory which, he argues, provides a better picture of rationality.¹ The second challenge pertains to risk-weighted expected utility. Within such a framework, a gambler can remain unexploitable although she breaks some of the laws formulated above. Here, however, the discussion is not pursued.

Likewise, the sixth section addresses an objection to the final Dutch Book arguments which, this time, concerns their third premise (DB3). It thereby offers an insightful reflection on the connection between exploitability, consistency, and rationality. After reviewing and refuting several arguments defending the claim that inconsistent preferences are irrational, Pettigrew develops a pragmatic argument for *Probabilism* (i.e., *Normalisation* and *Finite Additivity*) before extending it to the other laws of credence. In a nutshell, he shows that any non-probabilistic credence

¹ The disjunctive norm relies almost exclusively on expected utility theory. However, when considering a bet on two propositions, one with a 0% credence and another with a positive credence, DB1 should be used. This is so because in such cases expected utility theory, unlike DB1, could lead a gambler to select a dominated act.

function can be replaced by a probabilistic credence function providing greater utility. Hence, *Probabilism* better guides our gamblers.

The seventh section conducts a series of ‘robustness checks’ to evaluate Dutch Book arguments’ potential for generalisation, and finally, the eighth section compiles the mathematical proofs used throughout the book.

In brief, this concise book guides the reader nicely through the literature surrounding Dutch Book Arguments and presents interesting solutions to several issues. With a compelling overall view, some smaller points may appear less convincing. I highlight three below.

II. THE RELEVANCE OF DUTCH BOOK ARGUMENTS

If one subscribes to the view that a book must discuss its own relevance, the conciseness of *Dutch Book Arguments* might be gained at the expense of presenting its usefulness: why care about Dutch Book arguments in the first place? The book is mostly elusive about this question, even though Dutch Books have been used in diverse ways in the academic literature. One illustration of this can be found in probability theory. In this field, several established theories of probability adopt an *objective* interpretation of probabilities. Roughly put, this means that, when a flipped coin has a 50% probability of landing tails-up, this probability measures a *feature* of the world—this feature obtains independently of my beliefs about what the outcome will be. Nevertheless, objective theories of probability encounter difficulties that so far have not been resolved satisfyingly (see Resnik 1987 for a discussion).

Alternatively, probabilities can be interpreted *subjectively*. That is, a 50% probability that my coin will land tails-up reflects my *confidence*—i.e., my credence—in this statement. If, for example, I have reasons to doubt the fairness of the coin, the probability could vary. In short, credence shapes probability. Against this backdrop, Dutch Book arguments can help clarify how credence and probability (should) relate, thereby laying the ground for a deeper understanding of probability. More generally, Dutch Book arguments can be applied across decision theory for a variety of purposes (see Vineberg 2022 for an overview).

III. EPISTEMICALLY POSSIBLE WORLDS

Another point concerns possible worlds. As noted, Norma defines what credences she has given three possible worlds: *Low*, *Medium*, or *High*. But what does a ‘possible world’ amount to? In subsection 3.5, Pettigrew

defends the idea that the modality of possible worlds should be understood epistemically or logically, but not metaphysically. He reasons with the following example:

Southey is 10% confident in the proposition *I*, which says that Charlotte and Currer are the same person. Now, in fact, *I* is true. And so it is metaphysically necessary. But it is not logically necessary. And it is not epistemically necessary either, providing Southey does not have definitive evidence that Charlotte and Currer are the same person. [...] I propose the following: *W* is the set of all and only the worlds that are epistemically possible for you at the time in question. Thus, Southey is not dutchbookable in [this example], for while it is not metaphysically possible for Charlotte and Currer to be different people, it is epistemically possible for Southey, and thus a world in which *I* is false is an epistemically possible world for him. [...] In order to establish that someone is irrational, we have to show that their credences demand that they accept a series of bets that loses them money at every world that *for all they know* is the actual world. (21, 23)

But this reasoning is too quick. Consider again Norma. She, too, has to set her credences given three possible worlds, only one of which is metaphysically possible. However, she could argue that, *for all she knows*, the evidence she has does not justify raising her credences so much so that their sum equals 100%. Indeed, it remains open in her case to determine the benchmark against which a piece of evidence is compelling enough to raise her credences. She could for instance argue that, so far, her evidence only justifies a credence of 30% in each possible world, amounting to 90% in total. More would be epistemically unwarranted. On this argument, her credences appear epistemically rational although they do not obey *Normalisation*. Norma seems rational after all.

Still then, it could be retorted that this is a matter of logic: the proposition that global average temperature will either be *Low*, *Medium*, or *High* is simply a logical necessity. It does not matter whether Norma knows this: betting against a logical necessity is always irrational. In other words, the modality of possible worlds should be understood logically; if none is available (as is the case for Southey), then it should be understood epistemically.

But bringing in logical necessities might blur rather than clarify the issue, for logical and epistemic modalities may clash.² Consider the following situation: imagine that Norma invites all her colleagues to a restaurant on the first of June, the date of her birthday. Twenty-three people come (including herself). During a conversation with Reg, she learns that he, too, was born on a first of June. Surprised by this fact, they discuss how unlikely it must be, given this small gathering. Each of them takes a guess, but they disagree: Norma believes that the chances are below 30%, while Reg thinks they are higher. Eventually, he proposes to bet on the probability that, in this gathering, two of them share the same birth date. Norma quickly accepts (she bets that it is below 30%) before drawing her phone's calculator. The result is known as the Birthday Paradox: for only twenty-three people, the probability that two of them share the same birth date is in fact over 50%.

Now, was Norma irrational in accepting this bet? After all, she bet against a logical necessity, and so from a logical perspective she seems irrational. At the same time, she could not mentally calculate the probability quickly enough before accepting the bet. In other words, she did not know what the logical necessity was (nor did Reg). From an epistemic perspective, accepting the bet is rational. In sum, epistemic and logical modalities may clash. Their relation must be specified further.

IV. PREVIOUS BETS

My third and last point concerns previous bets (subsection 3.4). The claim Pettigrew defends is that previous bets matter: they can make it reasonable to accept a Dutch Book. He illustrates this point with Finella, a colleague of Norma's. Finella has a 30% credence in *Low* and a 40% credence in *Medium*. As a result, she sells two £100 bets, one for £35 on *Low* and one for £45 on *Medium*. Moreover, her credence in the disjunction *Low* \vee *Medium* is 90%. This means that she fails to abide by *Finite Additivity* (given that $30 + 40 = 70$). Based on this latter credence, she considers buying a third £100 bet on *Low* \vee *Medium* for £85. Here are her net gains given each possible world, dependent on whether she refuses or accepts the third bet:

² This issue arises because logical necessities are not always obvious; some necessitate a fair amount of cogitation. Such a critique is akin to that repeatedly raised against expected utility theory, as it assumes that individual agents can readily calculate and compare the utility of all prospects under consideration, even when this process requires complex computational power (Simon 1955).

	REFUSE	ACCEPT
LOW	$£(35 - 100 + 45) = -£20$	$£(35 - 100 + 45 - 85 + 100) = -£5$
MEDIUM	$£(35 + 45 - 100) = -£20$	$£(35 - 100 + 45 - 85 + 100) = -£5$
HIGH	$£(35 + 45) = £80$	$£(35 + 45 - 85) = -£5$

Table 1: Finella's Net Gains.

Intuitively, given Finella's credences, there is a Dutch Book against her: by accepting the new bet (which Finella is required to do given her credence in *Low v Medium*), she incurs a sure loss. So, it seems, she should refuse it. But Pettigrew argues that this is mistaken. First, he observes, Finella is in fact choosing between, on the one hand, a sure loss of £5 and, on the other, a bet on either a greater loss (-£20) or a large gain (£80). Finella's attitude towards risk is however not discussed here. Rather, Pettigrew suggests that her expected net gains are higher if she accepts the third bet (as it amounts to -£5) than if she refuses it ($[0.9 \times -£20] + [0.1 \times £80] = -£10$). She should then accept it. Therefore, he concludes, previous bets can make it rational to accept a Dutch Book.

To reach this conclusion, Pettigrew uses Finella's 0.9 (or 90%) credence in *Low v Medium* and infers that in *High* as equal to $1 - 0.9 = 0.1$. but since Finella breaks *Finite Additivity*, there exists another approach through which we can, assuming completeness, determine her 'full' credences. We could instead use her 0.3 credence in *Low*, her 0.4 credence in *Medium*, and infer that in *High* as equal to $1 - 0.3 - 0.4 = 0.3$. With these in mind, Finella's expected net gains are higher if she refuses the third bet ($[0.3 \times -£20] + [0.4 \times -£20] + [0.3 \times £80] = £10$) than if she accepts it (-£5), and so, clearly, she should refuse it. Given these two approaches, it may seem arbitrary to calculate the expected net gains of several bets using only a portion of the relevant credences.³

Pettigrew does not discuss this second approach because, in his view, Finella considers whether or not to accept the third bet; therefore, only her credence in *Low v Medium* should matter in calculating the expected gains of this bet (21). But Finella is not considering whether or not to accept the third bet *tout court*, she is considering whether or not to accept

³ This seems even more counterintuitive when we calculate Finella's expected gains in case she refuses the third bet, for in this case the expected gains amalgamate two bets (on *Low* and *Medium*) and a credence that initially played no role in accepting these bets.

it *given the previous two she has already accepted*. This is why we calculate here the expected gains not only of the last bet, but of all three. Yet the claim that Finella must use her credence in *Low v Medium* to determine whether the third bet is worth accepting does not imply that she must also use this credence to calculate the expected gains of all three bets. As such, Pettigrew seems to rely on an unstated assumption: that the credence(s) relevant for *present* bet(s) must also be used to calculate the expected gains of *both past and present bets*. Thus, no argument is provided for this assumption, on which the claim that ‘previous bets matter’ hinges.

In spite of this, *Dutch Book Arguments* makes numerous convincing cases, spanning a large array of related topics. If the book is of interest to academics across disciplines, it may also serve well those who enjoy leisurely gambling, and possibly future bookies.

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